

A Simulation Comparison of  
WinCross, SPSS, and Mentor Procedures  
for Estimating the Variance of a Weighted Mean

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Both WinCross and CfMC's Mentor provide unbiased estimates of the variance of the weighted mean. Therefore, to assess the difference between the WinCross estimate and the Mentor estimate of the variance of the weighted mean one must look at how far off each of these estimates of the variance can be from the true value of the variance of the weighted mean. An analytical expression of the magnitude of these discrepancies can be found on The Analytical Group, Inc. website: <http://www.analyticalgroup.com/download/WEIGHTED MEAN.pdf>.

To illustrate how the two estimates fared I created a simulated example, consisting of 1,000 samples of 100 observations from a normal distribution with mean  $\mu=1$  and variance  $\sigma^2=4$ . The weights I used were 100 random numbers from a uniform distribution between 0 and 1. (The value of  $f$  based on these 100 weights was 68.46327.) I applied these weights to each of the 1,000 samples of 100 observations and calculated the weighted sample mean from each sample. I thus had 1,000 weighted sample means, from which I could calculate an accurate estimate (based on 1,000 replicates) of the true variance of the weighted mean. The average of the 1,000 weighted means was 0.99702, and the variance was 0.05852.

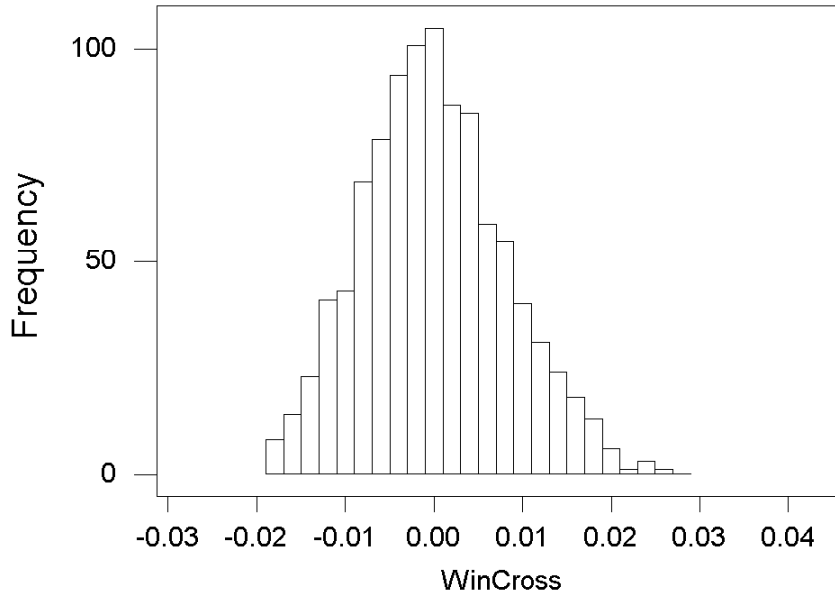
I now computed for each of the 1,000 samples the value of  $s^2/f$ , WinCross's estimate of the variance of the weighted mean, and  $s_c^2$ , Mentor's estimate of the variance of the weighted mean. The average value of WinCross's estimate was 0.05849 and that of the Mentor estimate was 0.05827.

By contrast, the average value of the SPSS estimate of the variance of the weighted mean was 0.09128. This large overestimate is a result of a slight bias in the estimate of  $\sigma^2$ , averaging 4.0202 (Mentor's unbiased estimate averaged 3.9892, WinCross's unbiased estimate average 4.0043) and the use of the divisor  $c=44.04058$  instead of  $f=68.46327$ .

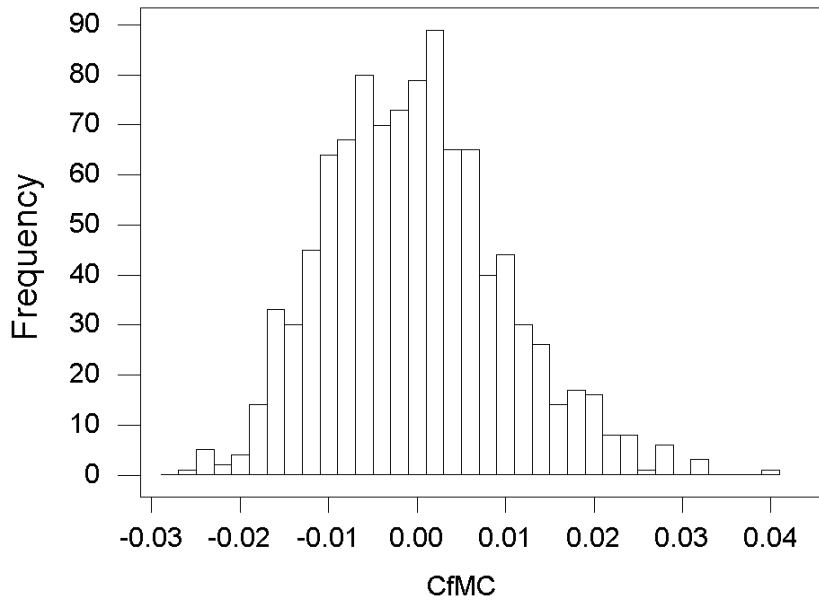
Of more importance is the fluctuation of the 1,000 variance estimates around the true value of 0.05852. The standard deviation of these 1,000 WinCross estimates was 0.00807 (quite close to the exact standard deviation, 0.008304), while that of the Mentor estimate was 0.01002 (quite close to the exact standard deviation, 0.0100369). This illustrates that the WinCross estimate of the variance of the weighted mean is more apt to be closer to the true value than is the Mentor estimate.

Following are histograms of the deviation of the 1,000 variance estimates from the true value for both the WinCross and Mentor estimates. Note that the distribution of 1,000 estimated variances using Mentor's method is more dispersed than is that based on the WinCross method.

Deviation of Estimated Variance from True Variance



Deviation of Estimated Variance from True Variance



This example illustrates that the WinCross method for estimating the variance of a weighted mean is preferable to that of Mentor.